**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS** : **We have a normal distribution with** ***μ* = 45 and *σ* = 8. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour, you must have X ≤ 50 so the question is to find P (X > 50).**

**P (X > 50)**

**= 1 - P (X ≤ 50).**

**Z = (X -*μ*)/*σ***

**= (X - 45)/8**

**Thus, the question can be answered by using the normal table to find**

**P (X ≤ 50)**

**= P (Z ≤ (50 - 45)/8)**

**= P (Z ≤ 0.625)**

**= 73.4%**

**Probability that the service manager will not meet his demand will be**

**= 100-73.4**

**= 26.6% or 0.266.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**ANS** : **We have a normal distribution with *μ* = 38 and** ***σ* = 6. Let X be the number of employees. So according to question**

**Probability of employees greater than age of 44= P (X>44)**

**P (X > 44)**

**= 1 – P (X ≤ 44).**

**Z = (X -*μ*)/*σ***

**= (X - 38)/6**

**Thus, the question can be answered by using the normal table to find**

**P (X ≤ 44)**

**= P (Z ≤ (44 - 38)/6)**

**= P (Z ≤ 1)**

**= 84.1345%**

**Probability that the employee will be greater than age of 44**

**= 100-84.1345**

**= 15.86%**

**So, the probability of number of employees between 38-44 years of age**

**= P (X<44)-0.5**

**= 84.13450.5**

**= 34.1345%**

**Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans** :

**Probability of employees less than age of 30 = P (X<30).**

**Z = (X -*μ*)/*σ***

**= (30 - 38)/6**

**Thus, the question can be answered by using the normal table to find**

**P (X ≤ 30)**

**= P (Z ≤ (30 - 38)/6)**

**= P (Z ≤ -1.333)**

**= 9.12%**

**So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400**

**= 36.48 (or 36 employees).**

**Therefore, the statement B of the question is also ‘TRUE’.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans** : **As we know that if X ~ N (µ1, σ1^2), and Y ~ N (µ2, σ2^2) are two independent random variables then,**

**X + Y ~ N (µ1 + µ2, σ1^2 + σ2^2), and**

**X − Y ~ N (µ1 − µ2, σ1^2 + σ2^2).**

**Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X and Y then,**

**Z ~ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).**

**Therefore, in the question**

**2X1~ N (2µ,4 σ^2) and**

**X1+X2 ~ N (µ +** **µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)**

**2X1-(X1+X2) = N (4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans: D.**

**The Probability of getting value between a and b should be 0.99.**

**So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e., 1-0.99).**

**The Probability towards left from a = -0.005 (i.e., 0.01/2).**

**The Probability towards right from b = +0.005 (i.e., 0.01/2).**

**So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.**

**By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.**

**Z= (X- μ) / σ**

**For Probability 0.005 the Z Value is -2.57 (from Z Table).**

**Z \* σ + μ = X**

**Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.5**

**Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.5.**

**So, the option ‘D’ is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:**

**Specify a Rupee range (centred on the mean) such that it contains 95 % probability for the annual profit of the company.**

**Mean Profit is Rs 540 million.**

**Standard Deviation is Rs 225.0 million**

**Range is Rs (99.00810347848784, 980.9918965215122) in Millions.**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans** **: 5th percentile of profit (in Million Rupees) is 170.0**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans :**

**Making 1 loss: - Confidence Interval: 0.0477903522728147.**

**Making 2 loss: Confidence Interval: 0.040059156863817086.**

**Probability of Division 1 making a loss in a given year is more than Division 2.**